Výsledky úloh

1. Úpravy výrazů

1.1. $|x+1|$, R^+ **1.2.** $1, a \neq 0, a \neq b$ **1.3.** $\frac{6}{a-b}, a \neq \pm b, a \neq -\frac{b}{2}$

1.4. $1+a+x, x \neq 0, x \neq a$ **1.5.** $\frac{a+2}{a^{n+1}}, a \neq 0; \pm 1; 2$

1.6. $15\sqrt[3]{x^2} - 3\sqrt[3]{9x^2}, x > 0$ **1.7.** $\sqrt[24]{\left(\frac{a+b}{a-b}\right)^{11}}, a \neq \pm b$

1.8. $\frac{\sqrt{m}\sqrt{n}}{\sqrt{m}+\sqrt{n}}, m \geq 0, n \geq 0, m \neq n$ **1.9.** $a, a > 0$

1.10. $\frac{x^2}{2x-1}, x \neq \frac{2}{3}; 0; \frac{1}{2}; 1; 2$ **1.11.** $\frac{2}{\sin x}, x \neq k\pi$ **1.12.** $2\operatorname{tg} x, x \neq k\frac{\pi}{2}$

2. Funkce

2.1. a) $D(f) = (2,3) \cup \langle 4,0)$ **b)** $D(f) = \langle -2,-1)$ **c)**

$D(f) = \left(-\infty, -\sqrt{2}\right\rangle \cup \left\langle \sqrt{2}, \infty\right)$ **d)**

$D(f) = \left\langle 2k\pi, \frac{\pi}{2} + 2k\pi \right\rangle \cup \left\langle \frac{3}{2}\pi + 2k\pi, \frac{5}{2}\pi + 2k\pi \right\rangle$ **e)** $D(f) = 2k\pi$ **f)**

$D(f) = \left(2k\pi, \frac{\pi}{2} + 2k\pi \right) \cup \left(\frac{3}{2}\pi + 2k\pi, \frac{5}{2}\pi + 2k\pi \right)$ **g)**

$D(f) = \left(-\infty, \frac{1}{2}\right\rangle \cup \left\langle \frac{2}{3}, \infty\right)$ **2.4.** $f^{-1} : y = \sqrt{\frac{x}{2}}, x \in \langle 2,32 \rangle$ **2.5. a)** < **b)** >

c) < **d)** < **e)** < **f)** > **2.6.** rostoucí pro $a \in (-\infty, -2)$, klesající pro $a \in (0, \infty)$

2.7. $D(f) = (-\infty, -1) \cup (-1, 2)$, rostoucí pro $s \in (-\infty, -1)$, klesající pro $a \in (-1, 2)$ **2.8.** $\log x = 3\log r - \log(2-r) + \frac{1}{3}\log g - \frac{1}{6}\log\left(t^2 - r^2\right)$

2.9 $t = 5{,}4\,\text{s}$, $s = 145{,}8\,\text{m}$ **2.10.** $\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$,

$\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$ **2.11.** 2 **2.12.** $\sin x = 0{,}6$; $\cos x = -0{,}8$; $\operatorname{tg} x = -\frac{3}{4}$, $\operatorname{cotg} x = -\frac{4}{3}$

3. Rovnice

3.1. $4e$ **3.2.** $-2{,}69$ **3.3.** $2;3; x < \sqrt[3]{35}$ **3.4.** $-0{,}99; 9$ **3.5.** $k\pi; \frac{\pi}{3} + k\pi$ **3.6.** $56°19' + k \cdot 180°; 33°41' + k \cdot 180°$ **3.7.** $49°47' + k \cdot 360°; 197°35' + k \cdot 360°$ **3.8.** $\frac{\pi}{4} + k\frac{\pi}{2}; \frac{2}{3}\pi + 2k\pi; \frac{4}{3}\pi + 2k\pi$ **3.9.** $k\pi; \frac{\pi}{4} + k\frac{\pi}{2}$ **3.10.** $\frac{\pi}{6} + 2k\pi; \frac{5}{6}\pi + 2k\pi$ **3.11.** $2k\pi; \frac{\pi}{2} + 2k\pi$ **3.12.** $\frac{5}{3}$ **3.13.** 4 **3.14.** -1 **3.15.** $\frac{3}{4}$ **3.16.** $-88; -24; 3$ **3.17.** 1 **3.18.** $-2-\sqrt{2}; -2+\sqrt{2}$ **3.19.** $a+1; b+1$ **3.20.** $20\ \mathrm{km \cdot h^{-1}}$

4. Rovnice s parametrem

4.1. $1; m \neq 0; m \neq \pm 1$ **4.2.** $\frac{a(2a-1)}{4a+1}; a \neq -1; -\frac{1}{4}; 0$ **4.3.** pro $k = -\frac{1}{2}, x \in R$; pro $k \neq \frac{1}{2}, x = 0$ **4.4.** $\frac{3a^2 - a - 2}{3a^2 + a - 2}; a \neq -1; 0; \frac{2}{3}$ **4.5.** pro $m \in (0;1)$ dva reálné kořeny**;** pro $m \in (-\infty;0) \cup (8;\infty)$ dva komplexně sdružené kořeny**;** pro $m \in \{0;8\}$ jeden dvojnásobný kořen

5. Soustavy rovnic

5.1. $[3;2;1]$ **5.2.** $\left[\frac{-25-18t}{11}; \frac{37+2t}{11}; t\right], t \in R$ **5.3.** $[1;1]$ **5.4.** $[t; 1-t]$ pro $a = 0;1$; $[0;1]$ pro $a \in R - \{0,1\}$ **5.5.** $\left[3; \frac{5}{2}\right]$ **5.6.** $m \in (-\infty; -3)$ **5.7.** $25{,}5\ \mathrm{m \cdot s^{-1}}; 17\ \mathrm{m \cdot s^{-1}}$ **5.8.** nemá řešení, rovnice určují hyperbolu a její asymptotu **5.9.** $I_1 = 1; I_2 = 2; I_3 = 3$

6. Nerovnice

6.1. $x \in \langle 1;0)$ **6.2.** $x \in (-\infty;2\rangle \cup \langle 4;\infty)$

6.3. $M_1 = (-3;7), M_2 = (-\infty;-4) \cup \langle -2;\infty)$ **6.4.** $x \in \left\langle -\frac{1}{4};\frac{1}{2} \right\rangle$ **6.5.**

$$x \in \bigcup \left[\left(\frac{\pi}{4} + k\pi; \frac{\pi}{3} + k\pi \right\rangle \cup \left(\frac{\pi}{2} + k\pi; \frac{5}{4}\pi + k\pi \right) \right]; k \in Z$$

7. Zobrazení

7.2. $v_z = 59{,}9\,\text{cm}; v_r = 66{,}5\,\text{cm}$ **7.4.** Dané body jsou středy stejnolehlosti obou kružnic **7.5.** 1

8. Rovinné obrazce

8.2. $t_a = 55{,}85\,\text{cm}; t_b = 36{,}64\,\text{cm}$ **8.3.** $r = \frac{a^2 + v^2}{2v} = 50\,\text{m}$

8.5. $a = 12\,\text{cm}; b = 8\,\text{cm}; c = 4\sqrt{13}\,\text{cm}$ **8.6.** $\alpha = 41{,}4°; \beta = 82{,}8°$

8.7. $v = 288\,\text{m}$ **8.8.** rovnoramenný **8.9.** $F_1 = 638{,}9N; F_2 = 620{,}5N$

8.10. $53°08'; 154°57'; 151°55'$ **8.11.** $1:2$ **8.12.** $675\,\text{cm}^2$ **8.13. a)** $o = 30; S = 30;$ strany $5;12;13$ **b)** $o = 24; S = 24;$ strany $6;8;10$ **8.14.** $v = 16\,\text{cm}; a = 40\,\text{cm}; c = 24\,\text{cm}$

8.15. $n = 5; S = 90{,}82\,\text{cm}^2; a = 10\,\text{tg}\,36°\,\text{cm} = 7{,}27\,\text{cm}$

9. Tělesa

9.1. $V = 2187\,\text{cm}^3$ **9.2.** $S = 2530\,\text{cm}^3; u = 35{,}4\,\text{cm}$

9.3. $a = 12\,\text{cm}; b = 5\,\text{cm}; c = 13\,\text{cm}$ **9.4.** $S = 547{,}06\,j^2$

9.5. $a = \sqrt[3]{3\sqrt{2}V}; S = \left(1 + \sqrt{3}\right) \cdot \sqrt[3]{18V^2}$

9.6. $v = \frac{\sqrt{6}}{3}a; V = \frac{\sqrt{2}}{12}a^3; S = \sqrt{3}a^2; \alpha = 54°$ **9.7.** $V = 507{,}8\,\text{cm}^3$

9.8. $S = 1400\,\text{cm}^2$ **9.9.** $v = \frac{r\sqrt{3}}{3}$ **9.10.** $x = \frac{3\sqrt{3}}{\sqrt[3]{2}}$ **9.11. a)** $h = 1\,\text{mm};$ **b)** $S = 31{,}4\,\text{mm}^2$ **9.12.** $V = \frac{226\pi}{3}\,\text{cm}^3; r = \sqrt{74}\,\text{cm}$ **9.13.**

$V = 1877\,\text{cm}^3; S = 1005\,\text{cm}^2$ **9.14.** zbývá $2\frac{1}{2}l$ vody **9.15.** $9:4$

9.16. $V = 98{,}6\,\text{cm}^3$

10. Analytická geometrie přímky a roviny

10.1. $\varphi = \frac{\pi}{3}$ **10.2.** $a_1 = 0; \alpha = 19°27'$ **10.3.** $\vec{v} = (24;-10); \vec{p} = (-24;10)$

10.4. rovnoramenný **10.5.** $\vec{n} = (2;3;4)$ **10.6. a)** $\frac{2-m}{3} = \frac{3n}{5} = \frac{3-q}{4}$

b) $\frac{2-m}{3} = \frac{3n}{5} \neq \frac{3-q}{4}$ **c)** $\frac{2-m}{3} \neq \frac{3n}{5}; q \in R$ **10.7.** čtverec neexistuje

10.8. pro $r \neq 0; m = 0{,}4; n = -0{,}8;$ pro $r = 0; \frac{3m-1}{2m} = \frac{3n+1}{3n}$ **10.9.** $a = -5$

10.10. $p: \frac{1}{7}x + y - 5 = 0; q: -7x + y + 45 = 0$ **10.11.** $v = 8{,}52$

10.12. různoběžné ; $P = \left[-\frac{2}{7}; \frac{5}{7}; 0\right]; \alpha = 65°55'$

10.13. $P_{xy} = [2;-3;0]; P_{yz} = [0;3;4]; P_{xz} = [1;0;2]$

10.14. $57x + 21y + 46z - 153 = 0$ **10.15.** $8x - 5y + z - 11 = 0$

10.16. různoběžné **10.17.** $v = \frac{10}{3}$ **10.18.** $v = 4{,}7$

10.19. průsečnice je přímka $x = 8 - t; y = t; z = 4$

10.20. $\alpha: x = -1 + u + v; y = -1 - 2u + 2v; z = 2 + u - 2v;$

$\alpha: 2x + 3y + 4z - 3 = 0$ **10.21.** $x + 2y + 1 = 0$

11. Analytická geometrie kuželoseček

11.1. $\alpha = 53{,}1°$ **11.2.** $v = 6{,}94$ **11.3.** $x^2 + y^2 - 18x - 12y + 92 = 0$

11.4. $(x-2)^2 + (y-4)^2 = 10$ **11.5.** $(x-5)^2 + (y-2)^2 = 10$

11.6. $E_1: \frac{x^2}{80} + \frac{4y^2}{5} = 1; E_2: \frac{x^2}{20} + \frac{y^2}{5} = 1$

11.7. $S = [3;-5]; a = \sqrt{10}; b = \sqrt{2}; e = 2\sqrt{2}$ **11.8.** $4x^2 + 9y^2 = 900$

11.9. $\frac{(x-3)^2}{26}+\frac{(y+6)^2}{25}=1$

11.10. elipsa ; $t_1: 2x-3y-9=0; t_2: 2x-3y-3=0$

11.11. $M=\left[-\frac{13}{18};-\frac{2}{3}\right]$ **11.12.** $S=108\sqrt{3}$ **11.13.** $|t|=\sqrt{522}$

11.14. $x^2=-40(y-90)$ **11.15.** $x-y-8=0$

11.16. $x^2-4y^2=12$ **11.17.** $\frac{x^2}{16}-\frac{(y-5)^2}{180}=1$

11.18. $S=[5,-3]; a=8; b=6; e=10; E=[15;-3]; F=[-5;-3]$

11.19. $9x^2-16y^2=144$ **11.20.** $v=\frac{3\sqrt{5}}{5}$ **11.21.** $\alpha=11{,}79°$

11.22. $t: 8x+3y+24=0$ **11.23.** $x^2-4y^2=12$ **11.24.** $T=\left[\frac{3}{2};15\right]$

11.25. $(x-13)^2+(y+19)^2=\left(9\sqrt{5}\right)^2$

12. Maticový počet

12.1. nelze **12.2.** $X=\begin{bmatrix}10 & 10 & -6\\ 6 & 2 & 10\\ 14 & 10 & -30\end{bmatrix}$ **12.3.** $X=\begin{bmatrix}1\\ 20\end{bmatrix}$ **12.4.** platí

12.5. $X=\begin{bmatrix}10 & -8 & 0\\ -\frac{8}{3} & \frac{22}{3} & 4\\ -\frac{32}{3} & -\frac{20}{3} & 10\end{bmatrix}$ **12.6.** $h(A)=2; |A|=4$ **12.7.** $x_1=2; x_2=3$

12.8. $a=0 \vee a=1$ **12.9.** $a\neq 5; [1-p-5q; 3p; -1+5p+7q; 3q]; p,q\in R$

12.10. nemá řešení $h(A)=3; h(\overline{A})=4$

13. Diferenciální počet

13.1. a) 6 **b)** $\frac{2}{3}$ **c)** 0 **d)** $\frac{\sqrt{11}}{22}$ **e)** $\frac{1}{3}$ **f)** -4 **g)** $\frac{1}{20}$ **h)** $\frac{\sqrt{2}}{2}$ **i)** 4 **j)** $\frac{1}{2}$ **k)** 8 **l)** $-\frac{\sqrt{2}}{2}$

13.3. a) $y' = -\frac{1}{x+x^2}$ **b)** $y' = \frac{ay - ay^3 + 2x}{3axy^2 - ax}$ **c)** $y' - 20x^{4x}\left(\ln x + 1\right)$ **d)** $y = \frac{2+x}{\left(x^2+1\right)\sqrt{x^2+1}}$ **e)** $y' = \frac{4x}{9\left(1-y\right)}$ **f)** $y' = \frac{1+3x^4}{x\left(1+x^4\right)}$ **g)** $y' = \frac{1}{\cos x}$ **h)** $y' = e^{\sqrt{\sin x}}\,\frac{\cos x}{2\sqrt{\sin x}} + 3\,\mathrm{tg}^2\,3x \cdot \frac{3}{\cos^2 3x}$ **i)** $y' = x^{\sin x}\left(\cos x \cdot \ln x + \frac{\sin x}{x}\right) + 2x\log e - \frac{1}{1+\cos x}$

13.4. $t : x + 2y - 5 = 0$ **13.5.** $M = \left[-\frac{3}{2};0\right]$ **13.6.** $\alpha = 60°; x = 8\,\mathrm{m}$ **13.7.** $r = 0{,}5\,\mathrm{dm}; v = \frac{4}{\pi}\,\mathrm{dm}$ **13.8.** lokální minimum $y(-1) = \frac{17}{12}; y(3) = -\frac{37}{4};$ lokální maximum $y(0) = 2;$ absolutní minimum $y(-2) = \frac{16}{3};$ absolutní maximum $y(3) = \frac{-37}{4}$ **13.9.** konvexní $\left(-\sqrt{3};0\right) \cup \left(\sqrt{3};\infty\right);$ konkávní $\left(-\infty;-\sqrt{3}\right) \cup \left(0;\sqrt{3}\right)$

13.11. $v(2) = 20\,\mathrm{m \cdot s^{-1}}; t = 4\,\mathrm{s}; s = 80\,\mathrm{m}; a = 10\,\mathrm{m \cdot s^{-2}}$

13.12. $t : 2x - y + 11 = 0; n : x + 2y - 7 = 0$ **13.13.** $s = \frac{1}{6}t(t-10)(t-16); v = \frac{1}{6}(t-4)(3t-40)$ **13.14. a)** $T = \left[-\frac{1}{2};-2\right]$ **b)** $T = \left[\frac{1}{2};1\right]$ **c)** $T = \left[-\frac{3}{2};-1\right]$ **13.15.** $t : x + y - 1 = 0; n : x - y + 1 = 0$

13.16. $R = R_i; P_{\max} = \frac{U^2}{4R_i}$

14. Integrální počet

14.1. a) $\frac{1}{2}x\left(\sin\ln x-\cos\ln x\right)+c$ **b)** $-\ln\left|\cos x\right|+c$ **c)** $-2\frac{\cos^6 x}{6}+c$ **d)** $\operatorname{tg}x-x+c$ **e)** $\frac{1}{2}e^x\left(\sin x-\cos x\right)+c$ **f)** $\sin^3 x+c$ **14.2. a)** $-\left(\frac{\cos\left[(a+b)x\right]}{a+b}+\frac{\cos\left[(a-b)x\right]}{a-b}\right)+c$ **b)** $\frac{2}{3}\sqrt{\operatorname{tg}^3 x}+c$ **c)** $\frac{x}{2}-\frac{1}{4}\sin 2x+c$ **14.3.** $c=\frac{1}{4}$ **14.4.** $F(x)=3x-x^2+\frac{x^4}{4}-\frac{1}{4}$ **14.5. a)** $S=\frac{27}{2}j^2$ **b)** $S=2{,}33j^2$ **c)** $S=36j^2$ **d)** $S=\left(\pi-\frac{2}{3}\right)j^2$

14.6. $V=\frac{2\sqrt{2}\pi}{3}j^3$

15. Kombinatorika a pravděpodobnost

15.1. a) 52 **b)** 18 **15.2.** $n=12$ **15.3.** 16 **15.4.** $n=7$ **15.5.** $n=7$

15.6. a) 120 **b)** 101 **15.7.** 30 **15.8. a)** $P=\{1-a;1+a\};x\geq 3$

b) $P=\{7\};y\geq 4$ **c)** $P=\{9\};x\geq 3$ **d)** $P=\{4\};x\geq 3$ **e)** $P=\{3\};x\geq 1$

f) $P=\{2;3\};x\in N$ **15.9.** neexistuje **15.10.** $x=1$ **15.11.** desátý člen

15.12. třetí reálný člen $840x^6;x_1=\frac{1}{2};x_2=-\frac{1}{2}$

15.13. $n=11;k=3;A_4=165$ **15.14. a)** $\frac{1}{2}$ **b)** $\frac{1}{2}$ **c)** $0{,}5667$

15.15. $P=0{,}078$ **15.16.** $P=0{,}306$ **15.17.** $P=0{,}6778$

15.18. $P=0{,}0638$ **15.19.** $P=0{,}465$ **15.20. a)** 36 **b)** 20 **c)** $0{,}1$

16. Posloupnosti a řady

16.1. $a_1=8-\frac{9}{2}d;d\in R$ **16.2.** $a=18\,\text{cm};b=24\,\text{cm}$

16.3. $20;18;16;14;12;10;8;6;4;2;0;-2;-4;-6;-8;-10;-12$ **16.4.** $o=72$

16.5. $a_1=10^4;q=10$ **16.6.** $n=5;q=3,a_1=18$ **16.7.** $1925\,\text{m}^3$ dříví

16.8. $s_8 = 510$ **16.9.** $75174\,\mathrm{m}^3$ dříví **16.10.** za 8 let **16.11. a)** $-6\left(1+\sqrt{3}\right)$

b) $4+3\sqrt{2}$ **c)** $\dfrac{n+1}{4}$ **16.12. a)** $x_1 = 1; x_2 = -1$ **b)** $P = \{6\}; x \neq 0; \dfrac{4}{3}$

17. Komplexní čísla

17.1. $z = -\dfrac{1}{10} - \dfrac{3}{10}i$ **17.2.** $A = 18+4i; \overline{A} = 18-4i; |A| = \sqrt{340}$ **17.3. a)**

$x = -\dfrac{1}{5}; y = \dfrac{6}{5}$ **b)** $x = -3; y = 2$ **17.4.** 0

17.5. $z = -\sqrt{3} - 3i; \bar{z} = -\sqrt{3} + 3i$ **17.6.** $a^5 = 4+4i$ **17.7. a)**

$z^n = 3^n\left(\cos n\dfrac{5}{3}\pi + i\sin n\dfrac{5}{3}\pi\right)$ **b)** $\dfrac{2+\sqrt{2}}{2} + \dfrac{2+\sqrt{2}}{2}i$ **17.8.** 64

17.9. $\cos 3\alpha = -\dfrac{117}{125}; \sin 3\alpha = \dfrac{44}{125}$ **17.10.** $-54+54i$ **17.11. a)**

$x_2 = \dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i; x_3 = -\dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i; x_4 = -1; x_5 = -\dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i;\ x_6 = \dfrac{1}{2} - \dfrac{\sqrt{3}}{2}i$

b) $x_1 = 3; x_2 = -\dfrac{3}{2} + \dfrac{3i\sqrt{3}}{2}; x_3 = -\dfrac{3}{2} - \dfrac{3i\sqrt{3}}{2}$ **c)**

$x_1 = \sqrt[5]{\dfrac{12}{35}}\left(\cos\dfrac{\pi}{5} + i\sin\dfrac{\pi}{5}\right); x_2 = \sqrt[5]{\dfrac{12}{35}}\left(\cos\dfrac{3\pi}{5} + i\sin\dfrac{3\pi}{5}\right);$

$x_3 = \sqrt[5]{\dfrac{12}{35}}(\cos\pi + i\sin\pi); x_4 = \sqrt[5]{\dfrac{12}{35}}\left(\cos\dfrac{7\pi}{5} + i\sin\dfrac{7\pi}{5}\right);$

$x_5 = \sqrt[5]{\dfrac{12}{35}}\left(\cos\dfrac{9\pi}{5} + i\sin\dfrac{9\pi}{5}\right)$ **d)** $x_1 = \cos 0 + i\sin 0;$

$x_2 = \cos\dfrac{2\pi}{5} + i\sin\dfrac{2\pi}{5}; x_3 = \cos\dfrac{4\pi}{5} + i\sin\dfrac{4\pi}{5}; x_4 = \cos\dfrac{6\pi}{5} + i\sin\dfrac{6\pi}{5};$

$x_5 = \cos\dfrac{8\pi}{5} + i\sin\dfrac{8\pi}{5}$ **e)** $x_1 = \sqrt[4]{5}(\cos 0 + i\sin 0);$

$x_2 = \sqrt[4]{5}\left(\cos\dfrac{\pi}{2} + i\sin\dfrac{\pi}{2}\right); x_3 = \sqrt[4]{5}(\cos\pi + i\sin\pi);$

$x_4 = \sqrt[4]{5}\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right); x_5 = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right);$

$x_6 = \sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right); x_7 = \sqrt{2}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right);$

$x_8 = \sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$; **17.12.** $x^6 + 64 = 0$

18. Výroky a výroková logika

18.1. a) $A \Rightarrow B; B \Rightarrow A$ **b)** $A' \Rightarrow B'; B' \Rightarrow A'$ **c)** $A \Rightarrow B; A' \Rightarrow B'$
d) $A \Rightarrow B; B' \Rightarrow A'$ **e)** $A' \vee B; A \vee B'$ **18.2.** je kontradikcí
18.3. platí:„Trojúhelník je rovnoramenný a není rovnostranný
18.4. $x \geq 5 \wedge x \leq 5; x = 5$ **18.5.** $(A \Rightarrow C) \wedge (B \Rightarrow C) \Rightarrow [(A \vee B) \Rightarrow C]$
je tautologie **18.6.** ano